UGANDA INSTITUTE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

END OF SEMESTER II EXAMINATIONS

DEPARTMENT	:	**ICT**
YEAR	:	**ONE**
SEMESTER	:	**TWO**
PROGRAM	:	**HIGHER EDUCATION CERTIFICATE (PHYSICAL)**
COURSE	:	**ADVANCED MATHEMATICS**
COURSE CODE	:	**HECE121**
DATE	:	**SUNDAY, JULY 7, 2024**
TIME	:	**09:00 AM – 12:00 NOON**
DURATION	:	**3 Hours**

INSTRUCTIONS

- The paper consists of two Sections A and B
- Answer **ALL** questions in Section A and **ANY THREE** questions in Section B
- Credit will be given to relevant examples and illustrations
- Don Not Write anything on this question paper

SECTION A (40 MARKS)

Attempt **ALL** the Questions in this Section.

1. The function $f(x) = x^3 + px^2 - 5x + q$ has a factor $(x-2)$ and has a reminder -5 when divided by $(x+3)$. Find the values of P and Q. **(4 Marks)**

2. Solve for X: $1 + \cos x = 2\sin^2 x$ for $0^0 \le x \le 360^0$. **(4 Marks)**

3. Determine the values of y for which the matrix $\begin{pmatrix} y & -3 \\ 1 & y-4 \end{pmatrix}$ is singular (**4 Marks)**

4. Show that $\frac{d}{dx}(\tan^{-1} x^x) = \frac{(1+Inx)x^x}{1+x^{2x}}$ (**4 Marks)**

5. Express $5+12i$ in polar form. (**4 Marks)**

6. Prove that: $\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right) = 2\tan 2\theta$ (**4 Marks)**

7. Given the information in the table below;

x	0.9	1.0	1.1	1.2
$f(x)$	0.260	0.242	0.218	0.192

Use linear interpolation to find;

i) The value of $f(1.04)$

ii) The value of x corresponding to $f(x) = 0.23$ (**4 Marks)**

8. Prove that $\int \tan\theta d\theta = In\sec\theta + c$ where c is a constant of integration. (**4 Marks)**

9. Given that $F = \begin{pmatrix} 1 & -2 & 7 \\ 6 & 0 & -1 \\ -3 & -10 & 4 \end{pmatrix}$. Determine the determinant of F **(3 Marks)**

10. Solve the different equation; $x(1-y)\frac{dy}{dy} + y = 0$. Given that $y = 1$ when $x = e$ **(5 Marks)**

SECTION B [60 MARKS]

Attempt **ONLY THREE** questions in this Section.

Question One:

a) Factorize the expression $2x^3+3x^2-32x+15$ completely. **(5 Marks)**

b) Express the function $f(x)=2x^2+8x+9$ in the form $2(x+a)^2+b$, and hence find the minimum value of the function. **(5 Marks)**

c) The expression ay^2+by+c is divisible by $(y-1)$, has a reminder 2 when divided by $(y+1)$ and a reminder 8 when divided by $(y-2)$. Find the value of a, b and c. **(5 Marks)**

d) Sketch the curve for $y=x^2(3-x)$ **(5 Marks)**

Question Two:

a) Solve the following equations for x;

i) $\sin 2x=\sqrt{3}\cos^2 x$ for $-180^0 \le x \le 180^0$ **(5 Marks)**

ii) $3\sin x+4\cos x=2$ for $-180^0 \le x \le 180^0$ **(5 Marks)**

b) Given that $x=\sec\mu+\tan\mu$ and $y=\sec\mu-\tan\mu$, by eliminating μ from the two equations, show that $xy=1$ **(4 Marks)**

c)

i) Prove that: $\dfrac{\cos A+CosB}{\text{sinA}+\text{sinB}}=\cot\dfrac{(A+B)}{2}$ **(3 Marks)**

ii) Deduce that: $\dfrac{\cos A+CosB}{\text{sinA}+\text{sinB}}=\tan\dfrac{c}{2}$ where A, B and C angles of a triangle (**3 Marks)**

Question Three:

a) Differentiate the following with respect to x

i) $(x+2)^2\sqrt{(x+1)}$ (**4 Marks)**

ii) $\dfrac{2x^2+3x}{(x-4)^2}$ (**4 Marks)**

b) Given that $y = (\sin x)^x$. Find $\frac{dy}{dx}$ **(4 Marks)**

c) If $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$, show that $\frac{dy}{dx} = \frac{1}{1-\sin x}$ **(8 Marks)**

Question Four:

a) Use Newton-Raphson iterative formular to show that the cube root of a number N is given by $\frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right]$ and hence taking $x_0 = 2.5$, determine $\sqrt[3]{10}$ and correct to 3 decimal places.
(8 Marks)

b) Use trapezium rule to estimate the $\int_1^{1.4} (x + \tan x)dx$ using 5 ordinates **(5 Marks)**

c) Given that $Z_1 = -1 + i\sqrt{3}$ and $Z_2 = -1 - i\sqrt{3}$

i) Express $\frac{Z_1}{Z_2}$ in the form $a + i\sqrt{b}$, where a and b are real numbers **(5 Marks)**

ii) Find $\left|\frac{Z_1}{Z_2}\right|$ **(2 Marks)**

Question Five:

a) Determine the following;

i) $\int xInxdx$ **(4 Marks)**

ii) $\int \sin^2 xdx$ **(4 Marks)**

b) Bacteria in a certain culture increases at a rate proportional to the number of bacteria present. If the number increases from 1000 to 2000 in 1 hour.

i) How many bacteria will be present after $\frac{3}{2}$ hours **(9 Marks)**

ii) How long will it take for the number of bacteria in a culture to become 4000? **(3 Marks)**

END