UGANDA INSTITUTE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY
WEEKDAY PROGRAMME END OF SEMESTER TWO EXAMINATIONS

ACADEMIC YEAR 2020/2021

DEPARTMENT: ICT

SEMESTER: TWO

PROGRAMME(S): DIPLOMA IN TELECOMMUNICATIONS ENGINEERING (DTE)
DIPLOMA IN ELECTRICAL & ELECTRONIC ENGINEERING (DEEE)

YEAR OF STUDY: ONE

COURSE: ENGINEERING MATHEMATIC I

COURSE CODE : MT121

DATE: THURSDAY 27TH, JANUARY 2022

TIME: 9:00 AM – 12:00 NOON

DURATION: 3 HOURS

INSTRUCTIONS:

(i) This paper contains two Sections: A (40 marks) & B (60 marks).

(ii) Attempt ALL questions in Section A, and ONLY THREE questions in Section B.

(iii) All questions in Section B carry equal marks.

(iv) Credit will be given for use of relevant examples and illustrations.

(v) Begin each number in Section B on a new page of the answer sheet.

(vi) DO NOT write on this question paper.

SECTION A [40 MARKS]
Attempt **ALL** the Questions in this Section.

1. State the **two** properties of random variable X defined by P(X= x). **(2 marks)**
   A discrete random variable X has a probability distribution given by $p(x)$, where $(X = x) = p(x)$, $\sum xp(x) = E(X)$, and E is the expectation of an event. If V is the variance of an event, prove that:
   (a) $V(X) = E(X^2) - \mu^2$
   (b) $V(aX + b) = a^2V(X)$ **[3 marks@] (6 marks)**
2. A manufacturer produces airmail envelopes whose weight is normal with Mean 1.95g and variance 0.000625g. The envelopes were sold in lots of 10,000. Find the number of envelopes in a lot:
   (a) heavier than 1.5 g
   (b) between 1.6 g and 2.1 g inclusive **[4 marks@] (8 marks)**
3. Solve the equation $y'' + 4y' + 5y = 13e^{3x}$, $y(0) = 2.5$ and $y'(0) = 0.5$. **(6 marks)**
4. (a) State the two standard conditions for a series to converge. **(2 marks)**
   (b) Using the conditions above, show that the series below converges.
   $$\sum_{n}^{\infty} \frac{1}{n^2(9n + 20)}$$
   (6 marks)
5. (a) State Taylor's theorem. **(2 marks)**
   (b) On which condition does Taylor's theorem become Maclaurin's series? Hence, find the expansion series for $f(t) = x^{-1}$ at $a = 1$ up to the 3rd term. **(4 marks)**
   (c) If $f(x)$ has Fourier coefficients a_n and b_n. Deduce $kf(x)$ has Fourier coefficients ka_n and kb_n. **(4 marks)**

SECTION B [60 MARKS]
Attempt **ONLY THREE** Questions in this Section.

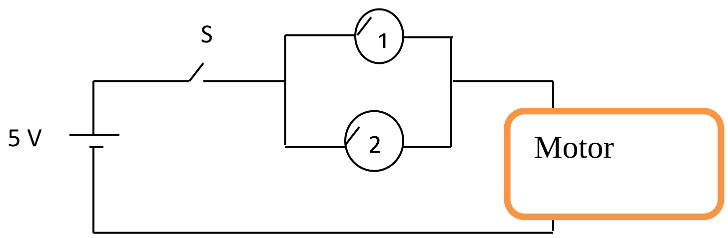
Question 1

(a) The field strength of a magnet B at a point on a circular axis, distance r from its centre, is given by $B = \frac{M}{2L}\left\{\frac{1}{(r-L)^2} - \frac{1}{(r+L)^2}\right\}$, where M is the moment and L is the half length of the magnet.
Using Taylor's expansion of series of your choice show that if L is very small compared to x, then $B \approx \frac{2M}{x^3}$. **(8 marks)**

(b) Given that $sin\, x = x - \frac{x^3}{3!} + \frac{x^5}{5!} -$ ... . Deduce the series expansion for $secx\ cos^3x$. **(12 marks)**

Question 2

The circuit below represents an equivalent electrical circuit of OR gate. The two relays 1 and 2 represent the inputs, and operate independently. When the switch S is on, the probability that a relay closes properly is 0.8. The motor runs once the circuit is complete.



(a) Develop a probability distribution table. **(4 marks)**

(b) Using the above, find:

(i) the probability that the motor runs **(2 marks)**

(ii) the expected value and standard deviation **(4 marks)**

(c) The relays operation is a random variable which is normally distributed. Find P(X $\leq$1.1), P(X≤ 2.5), P(-0.2 $\leq X \leq 1.2$), P(X $\geq$1.3) and P(X≤ -0.67).

[2 marks @] (10 marks)

Question 2

(a) The equation of motion of a body performing damped forced vibrations is given by $X'' + 5X' + 6X = Cost$, solve this equation given that X(0) =0.1 and X′(0) = 0. **(12 marks)**

(b) A type of bearing has an average life of 1500 hours and a variance of 1600 hours. Assuming a normal distribution, determine the number of bearings in a batch of 1200 likely to:

(i) fail before 1400 hours

(ii) last for more than 1550 hours **[4 marks @] (8 marks)**

Question 3

(a) Show that the series below converges

$$2 + \frac{3\times1}{2\times4} + \frac{4\times1}{3\times4^2} + \frac{5\times1}{4\times4^3} + \frac{6\times1}{5\times4^4} + \cdots$$ **(5 marks)**

(b) Find the Fourier series for;

$$f(t)=\begin{cases} t; & \frac{-\pi}{2} \leq t \leq \frac{\pi}{2} \\ \pi - t; & \frac{\pi}{2} \leq t \leq \frac{3\pi}{2} \end{cases}$$ **(15 marks)**

Question 4

(a) The thickness of 20 samples of steel plate is measured and the results in millimeters to two significant figures are tabulated below;

7.3	7.8	7.3	7.5
7.1	6.2	6.9	6.7
6.6	6.5	6.8	7.2
7.0	7.4	6.5	6.9
7.2	7.6	7.0	6.8

Divide a set of subscribers into regular classes of 0.2 mm. **(6 marks)**

(b) Using coding method, calculate:

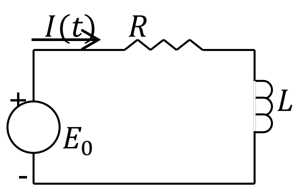
(i) mean (ii) variance (iii) standard deviation

of the steel thicknesses. **(14 marks)**

Question 5

(a) The electrical circuit below represents an application in lighting.

Find the general solution for $I(t)$ given that $I(0) = 0$.



Draw the waveform for current I against t and explain the graph. **(6 marks)**

(b) The table below shows a probability distribution function of a random variable:

X	0	1	2
p(x)	0.14	0.32	0.54

Find P(X≤1), P(X>1.5), P(X=0), P(X≥0), E(X), variance, standard deviation and F(2). **[2 marks @] (14 marks)**

END